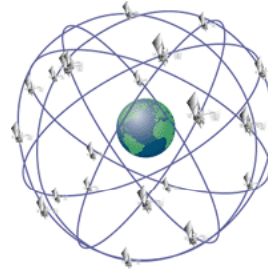
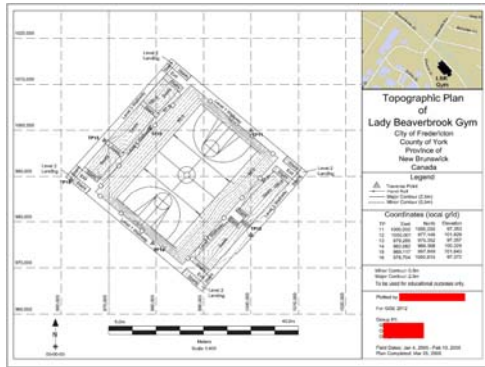


# Heighting

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# Heighting

## ◆ Leveling Errors

Table 5.1 Errors in Leveling

Source	Type	Cause	Remarks	Procedure to eliminate or reduce
Instrumental	Systematic	* Line of sight not parallel to axis of level tube	Error of each sight proportional to distance* ✖	Adjust instrument; <u>balance</u> sum of backsight and foresight distances
		Rod not standard length (throughout length) <sup>b</sup>	May be due to manufacture, moisture, or temperature; error usually small	<u>Standardize</u> rod and apply corrections, same as for tape
Personal	Random	* Parallax	...	Focus carefully
		Bubble not centered at instant of sighting	Error varies as length of sight	Check bubble before making each sight
		* Rod not held plumb	Readings are too large; error of each sight proportional to square of inclination* ✖	Wave the rod, or <u>use rod level</u>
Natural	Random	Faulty reading of rod or setting of target	...	Check each reading before recording; for self-reading rod, use fairly shot sights
		Faulty turning points	...	Choose definite and stable points
	Temperature variation	May disturb adjustment of level	Shield level from sun	
	Systematic	* Earth's curvature	Error of each sight proportional to square of distance* ✖	<u>Balance</u> each backsight and foresight distance; or apply computed correction
Random	* Variations in atmospheric refraction	Error of each sight proportional to square of distance* ✖	Same as for earth's curvature: also take <u>short sights</u> , well above ground, and take backsight and foresight readings in quick succession	
Pseudo-systematic <sup>c</sup>	Settlement of tripod or turning points	Observed elevations are too high	Choose stable locations; take backsight and foresight readings in quick succession preferably alternating order of sights	

\* The error of each sight is systematic, but the resultant error is the difference between the systematic error for foresights and that for backsights; hence, the resultant error tends to be random.

<sup>b</sup>Uniform wear of the bottom of the rod causes no error.

<sup>c</sup>The sign is always the same but the error is of an unknown magnitude.

(GGE1001 Lecture note)

## Barometric Heighting

### ◆ Overview

- Low cost, simple way to determine height differences
- Not very accurate
- Basics Principle:
  - as air pressure decreases with altitude, height differences can be derived from pressure differences
- Air pressure is measured by the barometer
- Problems:
  - air pressure is a *non-linear function of height*
  - pressure depends on temperature, humidity, geographic location, meteorological conditions, etc.



Visit our website, <http://kepler.gge.unb.ca>

(Dare, 2007)

3/00

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## Barometric Heighting

### ◆ Barometric Height Formula

- If one imagines a vertical cylinder with its base as unit area cut out of the atmosphere, the pressure at the base of the cylinder is equal to the weight of the column of air. At a point  $h + dh$  above the base of the cylinder the pressure is lower than for  $h$  by an amount equal to the weight of the column of air of height  $dh$ :

$$dp = -g\rho dh$$

where:  $g = 9.81 \text{ m/s}^2$  is the acceleration due to gravity,  $\rho$  = the density of air.

- Introducing the volume  $V$  of the gas, we obtain:  $\rho = \frac{M}{V}$   
 where  $M$  is the mass of the gas and, using the equation of state:

$$pV = nRT = nLkT, \quad dp = -p \frac{mg}{kt} dh$$

where  $n$  is the number of moles,  $T$  is the temperature in Kelvin,  
 $R = 8.31 \text{ JK}^{-1} \text{ mol}^{-1}$ , universal gas constant,  $L = 6.02 \cdot 10^{-23} \text{ mol}^{-1}$   
 Loschmidt number,  $k = 1.38 \cdot 10^{-23} \text{ JK}^{-1}$ , Boltzman constant,  $m$ =mass of molecule

4/00

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## Barometric Heighting

### ◆ Barometric Height Formula

- If the equation is solved for  $p$  for

$$p = p_o, \quad \text{where } h = 0$$

Then we obtain:

$$\therefore p = p_o e^{-\frac{mgh}{kT}} \rightarrow \frac{p}{p_o} = \frac{\rho}{\rho_o}$$

$$\therefore \rho = \rho_o e^{-\frac{mgh}{kT}}$$

So, the pressure is the non-linear function of height, and the barometric height  $h$  is dependent on: vapor pressure, temperature, surface pressure.

[http://www.fizika.org/skripte/of-prakt/1\\_4\\_07.pdf](http://www.fizika.org/skripte/of-prakt/1_4_07.pdf)

5/00

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## Barometric Heighting

### ◆ Accuracy of Barometric Heighting

- Accuracy of  $\Delta H$  depends mainly on:
  - accuracy of barometer readings, e.g. B1 and B2
  - accuracy of meteorological data,
- Mean altitude  $\Phi_m$  and mean height  $H_m$  are usually known accurately (i.e. they do not contribute to errors in  $\Delta H$ )
- Applying law of error propagation, we get

$$\sigma_{\Delta H} \approx 18,400 \log e \sqrt{\frac{1}{B_1^2} + \frac{1}{B_2^2}} \sigma_B$$

- If  $B_1 = B_2 \approx 1,000 \text{ mbar}$ ,  $\sigma_B = 0.1 \text{ mbar}$ , accuracy of a height difference of 1.2m is obtained.
- For best result, one or more control barometers are to be used.

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## cf. Effect of Refraction

### ◆ Formula for refraction

- As density of air decreases with altitude, optical rays get refracted towards the higher density layers, forming curved lines
- A refracted line can be approximated as a circular arc
- Experience has shown that the radius of the circular arc is about *Eight* times the radius of the earth, i.e.

$$r \approx 8R \rightarrow c_2 \cong \frac{S^2}{2r} \cong k \frac{S^2}{2R} \rightarrow c_2 \cong \frac{D^2}{2r} \cong k \frac{D^2}{2R} \text{ (from Fig.1)}$$

- Where,  $k = 1/8 \approx 0.13$  is called the refraction coefficient, whose actual value may vary by as much as  $\pm 0.04$  depending on the meteorological conditions

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## cf. Effect of Curvature

### ◆ Formula for curvature

- Leveling error due to the curvature:

$$R^2 + T^2 = (R + e_p)^2 \rightarrow e_p = (R^2 + T^2)^{1/2} - R = R + \frac{T^2}{2R} - \frac{T^4}{8R^3} - R$$

$$e_p \doteq \frac{T^2}{2R} \quad (T \ll R) \rightarrow \boxed{c_1 \doteq \frac{D^2}{2R} \quad (c_1 \equiv e_p, T \equiv D)}$$

- $c_1$ , and  $c_2$  is to be added or subtracted from the basic  $\Delta H$  equation, which lead to:

$$\Delta H = D \cdot \cot Z + I - T + (1 - k) \frac{D^2}{2R}$$

(final formula for trigonometric height corrected for refraction and curvature)

10/00

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## Trigonometric Heighting

### ◆ Examples

- 1) A person stands on a point at a beach. When he is 1.7m tall and the elevation of the point is 45.2m, how much of the distance can he see to the horizon (Assumption:  $R=6370\text{km}$ ,  $k=0.14$ , refraction coeff.)?

Ans.)

- 2) You're supposed to determine the height difference between A and B using trigonometric heighting. The distance between A and B is 5 km. How much do you have to adjust the errors due to the curvature and refraction in this case (Assumption:  $R=6370\text{km}$ ,  $k=0.14$ , refraction coeff.)?

Ans.)

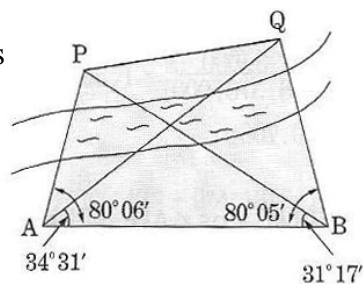
11/00

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## Trigonometric Heighting

### ◆ Examples

- 3) Due to the obstacle as shown in the figure, the distance between P and Q is supposed to be determined based on angular measurement only, e.g. using reflectless EDM. When the distance between A and B is 216.90m, what is the distance between P and Q?



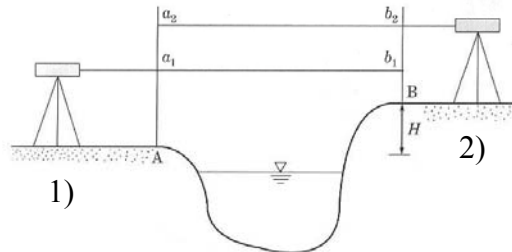
Ans.)

12/00

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## Simultaneous Reciprocal Heighting

- ◆ River or valley crossing where BS and FS distances can not be balanced, e.g. using Level



- reduce the collimation error, and the errors due to the curvature and refraction

$$H = \frac{(a_1 - b_1) + (a_2 - b_2)}{2}$$

13/00

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## Simultaneous Reciprocal Heighting

- ◆ River or valley crossing where BS and FS distances can not be balanced, e.g. using EDM

$$H_2 - H_1 = 2R \left( 1 + \frac{H_2 + H_1}{2R} \right) \frac{\tan \left\{ \left( z_2 - z_1 + \Delta z_2 - \Delta z_1 \right) / 2 \right\}}{\cot(\gamma/2)}$$

- Writing the terms:

$$\frac{2R}{\cot(\gamma/2)} = 2R \tan(\gamma/2) \approx S^H, \text{ and } (H_1 + H_2)/2 = H_m$$

- Assuming symmetric ray path, i.e.,  $\Delta z_1 = \Delta z_2$ , we get

$$\Delta H = S^H \left( 1 + \frac{H_m}{R} \right) \tan \left( \frac{z_2 - z_1}{2} \right) \quad \text{From [Fig.2]}$$

[Q] Fundamental difficulties in solving the above equation?

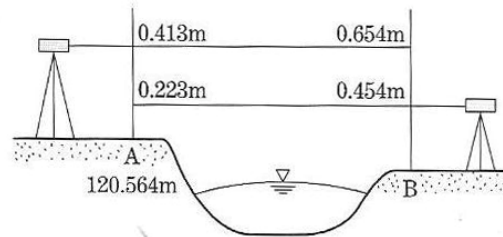
14/00

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## Simultaneous Reciprocal Heighting

### ◆ Example

- 1) Based on the simultaneous reciprocal heighting, the following values are achieved. When the elevation of the point A is 120.564m, what is the elevation of the point B?



Ans.)

15/00

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## GPS Heighting

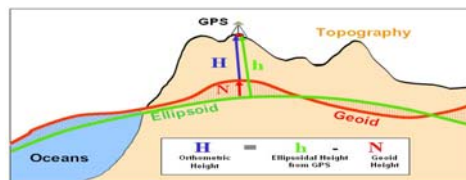
### ◆ GPS Height

- Geoid ellipsoidal separation (N):
- : Is the distance between the ellipsoid and geoid at a specific point and is also known as the Geoid Undulation. The separation added to the orthometric (sea level) height (H), results in the ellipsoid height (h). The following formulae is of particular importance with the use of GPS.

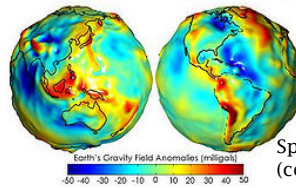
$$h = H + N$$

where

- h = is the ellipsoidal height (height usually obtained with GPS)
- H = is the orthometric height (sea-level)
- N = is the geoid ellipsoidal separation



(courtesy from NRCan)



Spherical harmonic representation (courtesy from wikipedia, units of gravity)

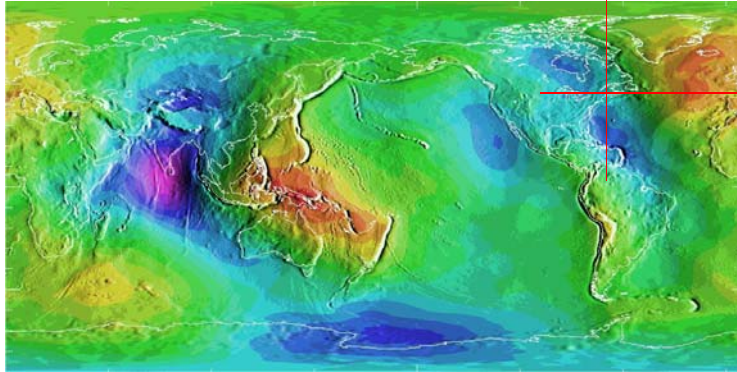
16/00

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## GPS Heighting

◆ **For height, how do we get N?**

- Global geoidal map (accuracy a few metres).
- National geoidal map (accuracy a few cms-dms).
- Establish local geoid model (accuracy a few cms).



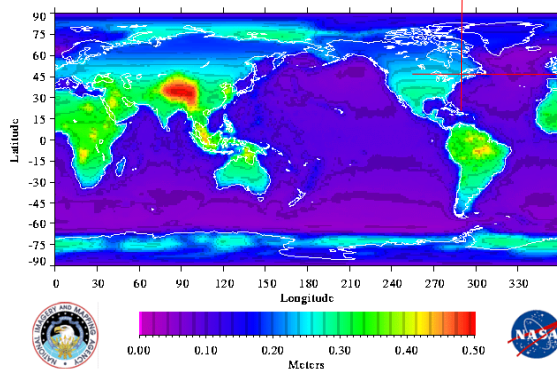
(Red: 85.4m or higher, Magenta: -107.0 meter or lower, from 15' x 15' undulation)  
Based on NIMA/GSFC WGS-84 EGM96 15' Geoid Height File  
(<http://164.214.2.59/geospatial/products/GandG/wgs-84/geos.html>)

## GPS Heighting

◆ **Geoid Height N**

$$N = r_E \sum_{n=2}^{\infty} \sum_{m=0}^n \bar{P}_{nm} (\sin \phi) (\bar{C}_{nm}^* \cos m\lambda + \bar{S}_{nm}^* \sin m\lambda)$$

EGM96 Geoid Error, 70x70



(<http://cddis.gsfc.nasa.gov>)

## GPS Heighting

### ◆ CGG2000 Model

- : A new and enhanced gravimetric geoid model **for Canada**
- : Replaced the GSD95 geoid model made by NRCan, UNB, and UofCalgary.
- : Follows the Helmert-Stokes scheme, i.e., the gravity measurements are reduced in terms of Helmert's second condensation method and the geoid heights are determined using the Stokes integral.
- : All related reductions to the gravity measurements are determined using the **spherical approximation**.
- : The under-laying global geopotential model is **EGM96 (degree and order 360)** and it contributes its long wavelengths up to degree and order 30 through the modified spheroidal Stokes kernel.
- : The 1D-FFT procedure solves the Stokes integral. CGG2000 model is validated against GPS/Levelling across Canada.
- : For 1090 benchmarks distributed across Canada, **the mean and standard deviation is -0.260 m and 0.179 m, respectively**. Part of the misfit is probably due to a systematic error in the Canadian primary levelling network.
- : the height represents the separation between geocentric reference ellipsoid of GRS80 and the equipotential surface.

(The Canadian Gravimetric Geoid Model of 2000 (CGG2000) by Marc Veronneau, NRCan)

19/00

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## GPS Heighting

### ◆ GPS Height

#### **GPS-H:**

- GPS Height Transformation Packages.
- [http://www.geod.nrcan.gc.ca/tools-ouils/gpsh\\_e.php](http://www.geod.nrcan.gc.ca/tools-ouils/gpsh_e.php)
- Online or Offline programs are provided. Offline program, called "gpsh-e.zip" released on Nov. 2001, is Win16 environment. Please check your computer to install it.

20/00

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# GPS Heighting

## ◆ GPS Height

Geoid Model:

CGG2000 - Canadian Gravimetric Geoid model 2000 is a scientific model of the geoid for North America from gravity data up to 2000.

21/00

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# GPS Heighting

## ◆ GPS Height

Online Service (from NRCan website)

[http://www.geod.nrcan.gc.ca/apps/gpsh/gpsh\\_e.php](http://www.geod.nrcan.gc.ca/apps/gpsh/gpsh_e.php)

Input coordinates manually

22/00

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# GPS Heighting

## ◆ GPS Height

Online Service (from NRCan website)

[http://www.geod.nrcan.gc.ca/apps/gpsh/gpsh\\_e.php](http://www.geod.nrcan.gc.ca/apps/gpsh/gpsh_e.php)

Input Data		Results	
Longitude Code:	west		
Reference System:	NAD83		
Latitude:	45 degrees 57 minutes 02.18748 seconds		
Longitude:	66 degrees 38 minutes 27.68119 seconds		
Ellipsoidal Height (m):	12.689		
Output Data			
Geoid Height (m):	-22.413		
Orthometric Height (m):	35.102		

This is an example result from the previous page.

Please check your results.