GGE2012 Advanced Surveying (Course Note)

Planning GPS Survey

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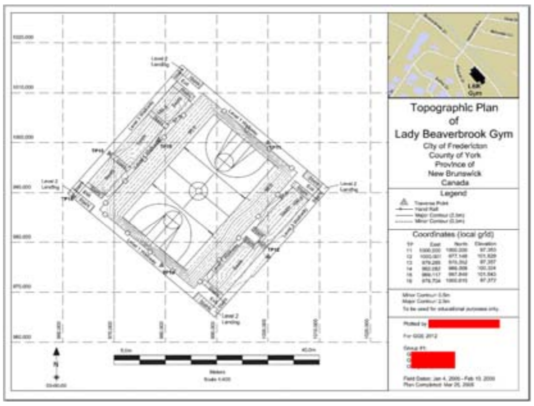
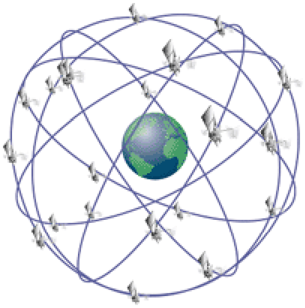




Planning GPS Survey

GPS Phase Eq.:

$$\Phi_r^s = \rho_r^s + c\cdot\delta t_r + c\cdot\delta t_{r,sys} - c\cdot\delta t^s - c\cdot\delta t^s_{sys} + \delta\rho_{trp} - \delta\rho_{ion} + \delta\rho_{rel} + \delta\rho_{mul}$$

$$+\lambda\cdot N_r^s + \ldots + \varepsilon$$

(unit: meter). More details for meaning each items: please see a GGE2012 note (later part of this notes) or a textbook (Kaplan, or Hoffman-Wellenhof Textbook etc.-notation is slightly differ.)

Short-Baseline, e.g. < 10 km

: Correlation of the atmospheric errors, orbital errors

: Multipath, Noise, (note: we will use a relative positioning)

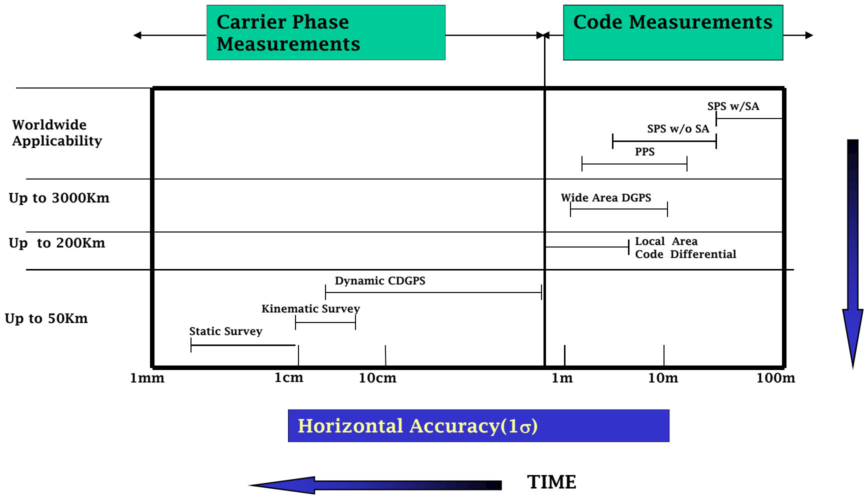
Long-Baseline, e.g. > 50 km

: De-correlation of the atmospheric errors, orbital errors

: Multipath, Noise

GPS (Global Positioning System)

◆ Methods vs. Accuracy



Planning GPS Survey

Critical Parameters for GPS Field Survey

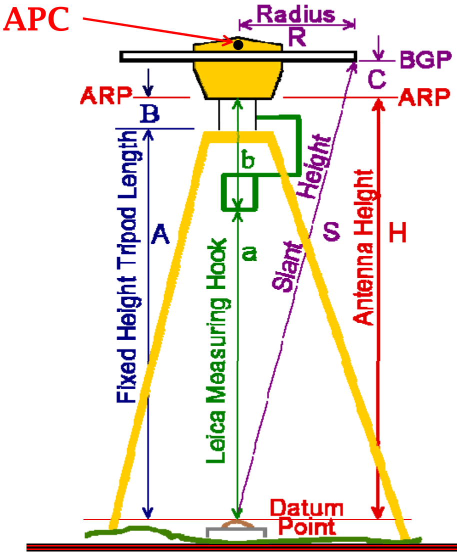
1. *Antenna Height – Very important!!!*
2. *Satellite Geometry (DOP, or Masking Angle)*
3. *Environmental Dependent Error Sources (Multipath and Cycle slip due to the Urban Canyon, Canopy; What others? Cables?)*
4. *Instrumental Failure (Pre-Examined)*
5. *Planning Instruction*

Planning GPS Survey

- *Example of GPS instrument*

Planning GPS Survey

1. Antenna Height



1. *The proper recordation of antenna height is **critical**. The Antenna Height used "at NGS" is the vertical distance between the station datum point and the Antenna Reference Point (ARP). Observers must carefully measure and check this height, and record and describe all measurements and antenna constants. Record all values to 0.0001 meters or 0.001 foot. All measurement computations must be checked and initialed by another person.*
2. Antenna Height $H = (sqrt(S^2 - R^2) - C)$
3. ***What's the common mistakes and what kind of heights is from GPS?***

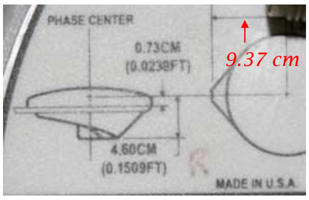
Reference (figure):
www.ngs.noaa.gov/PROJECTS/GPSmanual/observations.htm#antenna

Planning GPS Survey

1. Antenna Height (pl. use APC)

: Please calculate antenna phase center (APC) values provided on the back of the antenna.

: Zephyr™Antenna

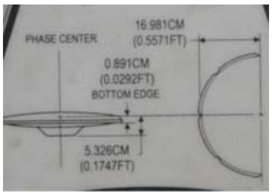


Planning GPS Survey

1. Antenna Height (pl. use APC)

: Please calculate antenna phase center (APC) values provided on the back of the antenna.

: Zephyr Geodetic™ Antenna



Planning GPS Survey

1. Antenna Height (pl. use APC)

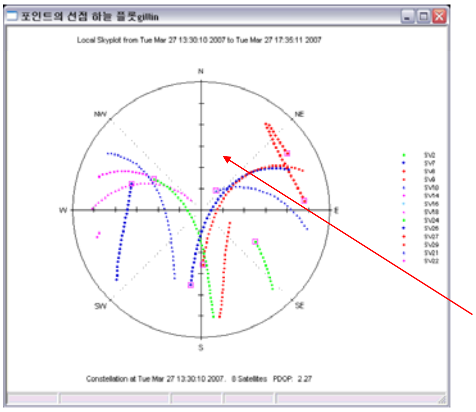
: Please calculate antenna phase center (APC) values provided on the back of the antenna.

: ***Micro Centered L1/L2***

Planning GPS Survey

2. Satellite Geometry

: Concerned about geometry and visibility?



[Check]

: SkyPlot

: Survey Plan

: DOP Check – Next Slide

*(**must be pre-determined**)*

- What is happened on this area?

- How can you select your area for your survey points based on this figure?

Planning GPS Survey

2. Satellite Geometry

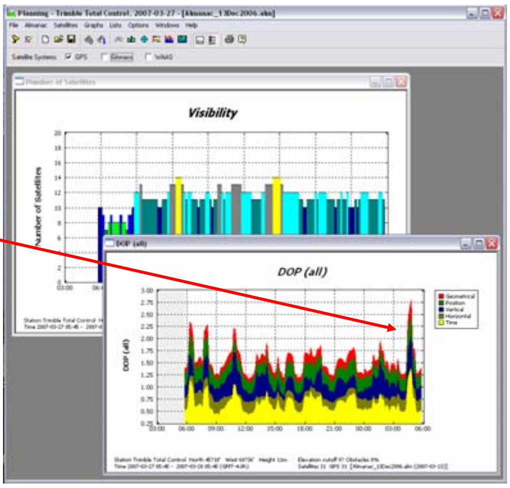
: Concerned about geometry and visibility?

*: Please use the “**Planning**” Tool before you start surveying from TGO™ !!!*

: Can you find any bad time for your survey schedule from the right figure?

: You can check them in any time and any location in the world if you provide the proper almanac to the software (it’s in the TGO utilities!)

: What is the masking angle? What is its usual value?

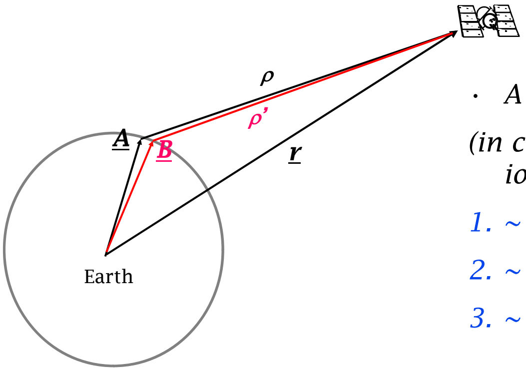


Planning GPS Survey

3. Environmental Dependent Errors

: Multipath, cycle slip and atmospheric effects

: What kind of environmental errors should be included on your processing for differential GPS? Think of GPS’s geometric range and your baseline length.



- *A (reference), B (rover)*

(in case of no severe trop. and iono. activity)

1. *~ 10 km baseline case*
2. *~ 50 km baseline case*
3. *~ 100 km baseline case*

Planning GPS Survey

4. Instrumental Failure

: Cable connection – Common failures

: Receiver failure – Unusual case, but have to examined before you survey.

: Battery failure – Usual case, have to make it sure before your field work.

You will be given a sheet whether you find any problems on your instruments during survey.

5. Next: Details of Math. Eq. of Carrier Phases

: You don't need to remember each term of the equations, but you "MUST" understand the characteristics of single and double differences.

Detailed Math. Eq. of Phases

Carrier Phase Observables

- **Phase Observable** is the difference between the received satellite carrier phase (as sensed by the receiver's antenna) and the phase of the internal receiver oscillator

- The characteristics of Carrier-Phase Observable
  - Doppler frequency shifts of the GPS signals (-5,000 Hz ~ 5,000Hz) arise due to the relative motion between the satellites and the receiver
  - The receiver's Phase Lock Loop(PLL) measures the time-varying function $\varphi_k(t) - \varphi^p(t)$ by shifting the receiver-generated $\varphi_k(t)$ to track the received $\varphi^p(t)$
  - The measurements process cannot account for the number of whole carrier waves, "***Ambiguity***" between the receiver and the satellite.
  - If there is a temporary blockage of the transmitted signal $\varphi^p(t)$, the receiver loss lock on the signal and there is the possibility that the receiver will miss some of the whole cycle change in $\varphi_k(t) - \varphi^p(t)$.
  - Once phase lock is regained, the fractional part of $\varphi_k(t) - \varphi^p(t)$ is again measured correctly, but the counter register might show an incorrect value
  - This is called "***Cycle Slip***"

Detailed Math. Eq. of Phases

Doppler Effects

- Doppler equation in a literature (e.g. Leick (1995))

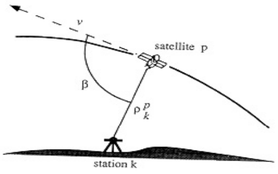
$$\frac{f^p}{f_T^p} = \frac{1-(v\cos\beta)/c}{\sqrt{1-v^2/c}} = \left(1-\frac{v}{c}\cos\beta\right)\left(1+\frac{v^2}{2c^2}+\frac{v^4}{8c^4}+\ldots\right)$$

where,

f^p ; received satellite frequency

f_T^p ; stable frequency emitted by the satellite

- The *topocentric range rate* is related to the tangential velocity v by



$$\dot{\rho} = \frac{d\rho}{dt} = -v\cos\beta$$

- The *ratio* and *difference* of the received and emitted frequency

$$\frac{f^p}{f_T^p} = 1 - \frac{\dot{\rho}_k^p}{c} \qquad f_T^p - f^p = \frac{f_T^p}{c}\dot{\rho}_k^p$$

Detailed Math. Eq. of Phases

Carrier Phase Equation

- Carrier phase observable $\varphi_k^p(t)$ for station *k* and satellite *p*

$$\varphi_k^p(t) = \varphi_k(t) - \varphi^p(t) + N_k^p(1) + I_{k,\varphi}^p(t) + \frac{f}{c}T_k^p(t)$$

$$+d_{k,\varphi}(t) + d_{k,\varphi}^p(t) + d_\varphi^p(t) + \varepsilon_\varphi$$

where,

$\varphi_k(t)$; receiver phase at the nominal reception time t

$\varphi^p(t)$; received phase at the nominal reception time t

$N_k^p(1)$; initial integer ambiguity

$\Rightarrow$ arbitrary counter setting of the tracking register
at the start of observation(phase lock)

$I_{k,\varphi}^p(t)$; ionospheric effect

$T_k^p(t)$; tropospheric effect

$d_{k,\varphi}(t), d_\varphi^p(t)$; hardware delay of receiver and the satellite

$d_{k,\varphi}^p(t)$; multipath effect

ε_φ; random carrier phase measurement noise

Detailed Math. Eq. of Phases

Carrier Phase Equation

- The difference $\varphi_k(t)-\varphi^p(t)$ is developed for a vacuum.
  - The basic idea of the carrier phase equation is the equivalence of the received carrier phase and the emitted phase at the satellite, exactly τ_k^p seconds earlier. So,

$$\varphi^p(t)=\varphi_T^p(t-\tau_k^p)$$

  - Consider the clock errors,

$$\varphi_k(t_r)=\varphi_k(t)+fdt_k,\qquad \varphi_T^p(t_r-\tau_k^p)=\varphi_T^p(t-\tau_k^p)+fdt^p$$

then,

$$\varphi_k(t)=\varphi_k(t_r)-fdt_k,\qquad \varphi^p(t)\equiv\varphi_T^p(t-\tau_k^p)=\varphi_T^p(t_r-\tau_k^p)-fdt^p$$

  - Finally, we get the carrier phase observable like as

$$\varphi_k^p(t)=\varphi_k(t_r)-\varphi_T^p(t_r-\tau_k^p)-fdt_k+fdt^p+N_k^p(1)+\varepsilon_T$$

  - The value of τ_k^p is about 70 msec, therefore the expansion of $\varphi_T^p(t-\tau_k^p)$ requires that the satellite frequency be modeled.

$$\dot{\varphi}_T^p(t_r)=f+a^p+b^p t$$

  - where a^p and b^p denote the satellite frequency offset and drift at emission time

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Detailed Math. Eq. of Phases

Carrier Phase Equation

- Since,

$$\varphi_T^p(t_r-\tau_k^p)=\varphi_T^p(t_r)-\int_\tau \dot{\varphi}_T^p(t)dt=\varphi_T^p(t_r)-[f+a^p+\frac{1}{2}b^p\tau_k^p]\tau_k^p$$

- &

$$\varphi_k^p(t)=\varphi_k(t_r)-\varphi_T^p(t_r)-fdt_k+fdt^p+\left(f+a^p+\frac{1}{2}b^p\tau_k^p\right)\tau_k^p+N_k^p(1)+\varepsilon_T$$

- After deleted the ignorable terms;

$$\varphi_k^p(t)=-fdt_k+fdt^p+(f+a^p)\tau_k^p+N_k^p(1)+\varepsilon_T$$

- The relation between the signal travel time and the topocentric range is

$$\tau_k^p=\frac{\rho_k^p(t)+\dot{\rho}_k^p(t)dt_k}{c}$$

- The final form of the carrier phase equation is

$$\varphi_k^p(t)=\frac{f}{c}\rho_k^p(t)-f\left[1-\frac{\dot{\rho}_k^p(t)}{c}\right]dt_k+fdt^p+N_k^p(1)+\frac{a^p}{c}\rho_k^p(t)$$

$$+I_{k,\varphi}^p(t)+\frac{f}{c}T_k^p(t)+d_{k,\varphi}(t)+d_{k,\varphi}^p(t)+d_\varphi^p(t)+\varepsilon_\varphi$$

Detailed Math. Eq. of Phases

Carrier Phase Equation (characteristics)

- Consider the carrier phase equation

$$\varphi_k^p(t) = \frac{f}{c}\rho_k^p(t) - f\left[1 - \frac{\dot{\rho}_k^p(t)}{c}\right]dt_k + fdt^p + N_k^p(1) + \frac{a^p}{c}\rho_k^p(t) + I_{k,\varphi}^p(t) + \frac{f}{c}T_k^p(t) + d_{k,\varphi}(t) + d_{k,\varphi}^p(t) + d_\varphi^p(t) + \varepsilon_\varphi$$

  - Large term of station clock error(fdt_k);
    - If station clock error is 1 nsec, then this term contribute 1.5cycles(150 times the expected accuracy).
    - For a phase measurement accuracy of 0.01 cycles, the required receiver clock accuracy is about 0.01 nsec.
  - Small term of station clock error(Doppler term)
    - Assuming $\left|\dot{\rho}_k^p(t)\right|$ < 800 m/sec & station clock error is 1 μsec, Doppler term contributes 0.004 cycles
    - If the station clock error does not exceed 0.1 μsec then the $\dot{\rho}$ is negligible.
  - Small term of satellite clock error(frequency offset)
    - Depends on the travel time of the signal to traverse the topocentric distance
  - Fortunately, most of error terms are either eliminated or their impact is significantly reduced by differencing techniques

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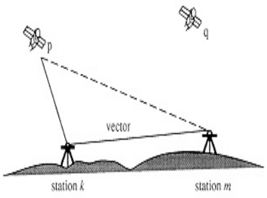
Detailed Math. Eq. of Phases

Single Difference

- The single-difference phase observable
  - If two receiver *k* and *m* observe the same satellite *p*

$$\begin{aligned}\varphi_{km}^p(t) &\equiv \varphi_k^p(t) - \varphi_m^p(t)\\ &= \frac{f}{c}\left[\rho_k^p(t) - \rho_m^p(t)\right] + \frac{a^p}{c}\left[\rho_k^p(t) - \rho_m^p(t)\right]\\ &\quad + \frac{f}{c}\left[\dot{\rho}_k^p(t)dt_k - \dot{\rho}_m^p(t)dt_m\right] - f(dt_k - dt_m) + N_{km}^p(1)\\ &\quad + I_{km,\varphi}^p(t) + \frac{f}{c}T_{km}^p(t) + d_{km,\varphi}(t) + d_{km,\varphi}^p(t) + \varepsilon_{km,\varphi}^p\end{aligned}$$



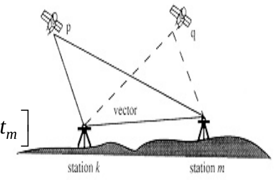
  - The principal advantage of the single-difference observation is that most of the errors common to the satellite cancel(satellite clock, hardware delay, ..)
  - The remaining small term converges toward zero.
  - However, the single-difference observations remain sensitive to both receiver clock errors dt_k and dt_m

Detailed Math. Eq. of Phases

Double Difference

- The double-difference phase observable
  - If two receiver *k* and *m* observe two satellite *p* and *q*

$$\begin{aligned}\varphi_{km}^{pq}(t) &\equiv \varphi_{km}^{p}(t)-\varphi_{km}^{q}(t)\\ &=\frac{f}{c}\left[\rho_k^p(t)-\rho_m^p(t)\right]-\frac{f}{c}\left[\rho_k^q(t)-\rho_m^q(t)\right]\\ &+\frac{a^p}{c}\left[\rho_k^p(t)-\rho_m^p(t)\right]-\frac{a^p}{c}\left[\rho_k^q(t)-\rho_m^q(t)\right]\\ &+\frac{f}{c}\left[\dot\rho_k^p(t)dt_k-\dot\rho_m^p(t)dt_m\right]-\frac{f}{c}\left[\dot\rho_k^q(t)dt_k-\dot\rho_m^q(t)dt_m\right]\\ &+N_{km}^{pq}(1)+I_{km,\varphi}^{pq}(t)+\frac{f}{c}T_{km}^{pq}(t)+d_{km,\varphi}^{pq}(t)+\varepsilon_{km,\varphi}^{pq}\end{aligned}$$



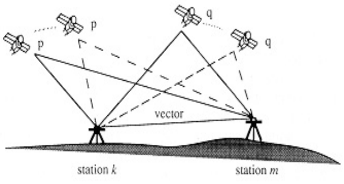
  - The principal advantage of the double-difference observation is that large receiver clock errors dt_k and dt_m are eliminated.
  - The integer ambiguity plays an important role in double differencing
  - Correlation must be taken consideration (math. one at least)

Detailed Math. Eq. of Phases

Triple Difference

- The triple-difference phase observable
  - Difference of two double differences between different epochs

$$\begin{aligned}\varphi_{km}^{pq}(t_2,t_1) &= \varphi_{km}^{pq}(t_2)-\varphi_{km}^{pq}(t_1)\\ &=\left[\varphi_{km}^{p}(t_2)-\varphi_{km}^{q}(t_2)\right]-\left[\varphi_{km}^{p}(t_1)-\varphi_{km}^{q}(t_1)\right]\\ &=\left[\varphi_{km}^{p}(t_2)-\varphi_{km}^{p}(t_1)\right]-\left[\varphi_{km}^{q}(t_2)-\varphi_{km}^{q}(t_1)\right]\\ &=\left[\varphi_{k}^{p}(t_2)-\varphi_{m}^{p}(t_2)\right]-\left[\varphi_{k}^{p}(t_1)-\varphi_{m}^{p}(t_1)\right]\\ &-\left[\varphi_{k}^{q}(t_2)-\varphi_{m}^{q}(t_2)\right]-\left[\varphi_{k}^{q}(t_1)-\varphi_{m}^{q}(t_1)\right]\\ &\ldots\ldots\end{aligned}$$



  - The advantages of the triple-difference observation are that the cancellation of the initial integer ambiguity $N_{km}^{pq}(t)$ and cycle slip detection.
  - The triple-difference solution is often considered a pre-processing techniques to get good approximate position for the double-difference solution. Also correlated.

Detailed Math. Eq. of Phases

Advantages of using Differencing Techniques

- In relative positioning, the common portion for both stations is automatically eliminated during the double-differencing operation
- **The characteristics of the differencing techniques**

Observation	**Effects eliminated**	Effects reduced	Option
Single difference	**first order Satellite clock**	orbit errors geometric position error ionosphere troposphere	constrain ambiguity
Double difference	**first order satellite & station clocks**		constrain ambiguity
Triple difference	**first order satellite & station clocks**		ambiguity eliminated

- Because of the cancellation of most of the effects of the propagation media and the clock errors, relative positioning has become so popular and useful in surveying.
- *What are the disadvantages of each methodologies?*